Interaction Between the Neutron Magnetic Moment and the Nuclear Coulomb Field in Neutron Polarization*

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Neutrons scattered through small angles by heavy nuclei are polarized by the interaction between the magnetic moment of the neutron and the electric field of the nucleus. Of the several approximate methods that have been used to estimate the magnitude of this effect, none are sufficiently general to permit the simultaneous consideration of a realistic nuclear potential that includes a spin-orbit interaction. Therefore such estimates are valid only for small angles of scatter ($\lesssim 5-10°$). A more nearly exact calculation based on an optical-model potential that includes a spin-orbit term is described in this paper. The calculation is based on a generalization of the usual Born approximation. This "generalized" method can be applied to a variety of problems in which the scattering potential is separable into a strong short-range term and a relatively weak long-range one. The results of this calculation are compared with data that originally indicated the possibility of an extranuclear contribution to the polarization of ~1.0-MeV neutrons scattered through an angle of 24°. This comparison indicates that the electromagnetic interaction can account for a substantial part of the polarization observed at this "large" angle—even for neutrons scattered from nuclei with moderate charge ($Z \gtrsim 40$).

I. INTRODUCTION

THE polarization of neutrons scattered from nuclei through "large" angles can be explained by the inclusion of an effective spin-orbit interaction in an optical-model potential. On the other hand, Schwinger[1] has shown that the polarization of neutrons scattered through small angles ($\lesssim$5–10°) is caused principally by the interaction that arises from the motion of the neutron magnetic moment in the nuclear Coulomb field.

To date this interaction has been considered mainly as a possible mechanism for the production of polarized neutrons. Consequently, Schwinger, in his original calculation of the polarization that results from this electromagnetic interaction, as well as Sample[2] and Baz,[3] in subsequent calculations, are concerned only with small angles of scattering. However, recent measurements[4] of ~1.0-MeV neutrons scattered from several nuclei with charge numbers in the neighborhood of $Z=40$ indicate that this interaction may contribute to the polarization at a scattering angle of 24°. In these measurements the corresponding differential scattering cross sections did not exhibit any anomalous behavior at 24° nor did the polarizations observed at the other angles (56°, 86°, 118°, and 150°). When we attempted to investigate the possibility that the interaction between the magnetic moment of the neutron and the Coulomb field of the nucleus is responsible for the abnormal polarization observed at 24°, we found that none of the approximate methods[1–3] used previously to estimate the magnitude of this effect could be extended meaningfully to scattering angles greater than 5–10°. For larger scattering angles it is necessary to consider simultaneously the polarization that arises from specifically nuclear forces. The details and results of such a calculation are presented in this paper.

The present calculation is based on a generalization of the usual Born approximation. This "generalized" approximation is applicable to any scattering problem in which the interaction potential can be separated into a short-range plus a relatively weak long-range term. This approximation is described in Sec. II. In Sec. III the method is used to calculate the polarization of neutrons scattered from an optical-model potential (that may include a spin-orbit term) plus the potential that describes the interaction between the magnetic moment of the neutron and the Coulomb field of the target nucleus. In Sec. IV the results of this calculation are compared with the polarization data of Ref. 4. This comparison shows that the electromagnetic interaction can account for a substantial part of the polarization observed at angles as "large" as 24°—even for neutrons scattered from nuclei with moderate charge.

II. THE BORN APPROXIMATION

We consider the scattering of neutrons from a spherically symmetric potential $V(r)$ that can be written in the form

$$V(r)=V_1(r)+V_2(r)\,, \tag{1a}$$

where

$$V_1(r)=0 \quad \text{for} \quad r\geqslant r_c\,, \tag{1b}$$

and

$$V_2(r)=0 \quad \text{for} \quad r<r_c\,. \tag{1c}$$

Both $V_1(r)$ and $V_2(r)$ may contain a spin-orbit term. It is assumed that the "cutoff" radius r_c in Eqs. (1) can be chosen such that $V_2(r)$ can be treated as a perturbation.

The usual generalization of the Born approximation consists in the following. The scattering problem is

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[1] Julian Schwinger, Phys. Rev. **73**, 407 (1948).
[2] J. T. Sample, Can. J. Phys. **34**, 36 (1956).
[3] A. I. Baz, Zh. Eksperim. i Teor. Fiz., **31**, 831 (1956) [English transl.: Soviet Phys.—JETP **4**, 704 (1957)].
[4] A. J. Elwyn, R. O. Lane, A. Langsdorf, Jr., and J. E. Monahan, Phys. Rev. **133**, B80 (1964).

solved exactly for some neighboring potential $V'(r)$ and the difference $\Delta V(r)$, where

$$\Delta V(r)=V(r)-V'(r)\,, \qquad (2)$$

is treated as a perturbation. To evaluate the integrals in the resulting Born expansion for the perturbed wave function, it is necessary that the unperturbed wave function be known at all points in space. In practice this usually means that a potential $V'(r)$ is chosen such that the corresponding stationary-state wave function can be expressed in terms of elementary functions. This procedure seldom leads to a potential $\Delta V(r)$ that can be treated as a perturbation.

In the method described below this difficulty can be avoided provided that the values of the phase shifts for scattering from the potential $V_1(r)$ are known. In the general case, these phase shifts can be evaluated only by a numerical integration of the wave equation with potential $V_1(r)$ so that the question naturally arises: "Why not solve the entire problem numerically?" For the type of problem we have in mind, the magnitude of the radius r_c in Eqs. (1b) and (1c) is of the order of the range of nuclear forces whereas the potential $V_2(r)$ extends to distances of the order of the radius of electronic orbits. In this case the usual partial-wave expansion, which for short-range potentials is ideally suited to numerical techniques, is much less appropriate. The inclusion of a potential $V_2(r)$ that extends to atomic dimensions has the effect that the interval over which the radial equation for each partial wave must be integrated is increased by several orders of magnitude. Furthermore, the number of partial waves that contribute significantly to the scattering is greatly increased. Under these conditions, rather elaborate provisions to avoid round-off errors in a numerical calculation are often necessary. Thus a numerical solution of the entire problem may be considerably more difficult than the numerical solution for the short-range potential $V_1(r)$ alone.

We now consider a Born expansion that is considerably better suited to scattering problems of the type described above. Let $\psi_{lj}(r)$ denote the l, jth radial function in a partial-wave expansion of the solution of the Schrödinger equation with a potential $V(r)$ as defined in Eqs. (1). This function satisfies the integral equation

$$\psi_{lj}(r)=j_l(kr)-ik\int_0^\infty dx\, x^2U_{lj}(x)\psi_{lj}(x) \times j_l(kx_<)h_l^{(1)}(kx_>)\,, \qquad (3)$$

where

$$h_l^{(1)}(kr)=j_l(kr)+in_l(kr)\,. \qquad (4)$$

Here j_l and n_l are the usual spherical Bessel and Neumann functions, respectively; U_{lj} is the potential that results from the operation of $2mV/\hbar^2$ on the spin and angular part of the l, jth wave function; $k^2=2mE/\hbar^2$, where E is the center-of-mass energy of the incident neutrons; $x_<$ is the lesser of x and r; and $x_>$ is the greater of x and r. If the value of ψ_{lj} at the radius $r=r_c$ is expressed as

$$\psi_{lj}(r_c)=A_{lj}(r_c)j_l(kr_c)+B_{lj}(r_c)n_l(kr_c)\,, \qquad (5)$$

the integral equation (3) can be written in the form

$$\psi_{lj}(r)=j_l(kr)\left[A_{lj}(r_c) -k\int_{r_c}^r dx\, x^2U_{lj}(x)n_l(kx)\psi_{lj}(x)\right]+n_l(kr) \times\left[B_{lj}(r_c)+k\int_{r_c}^r dx\, x^2U_{lj}(x)j_l(kx)\psi_{lj}(x)\right]. \qquad (6)$$

Substitution of the approximation

$$\psi_{lj}(r)\approx A_{lj}(r_c)j_l(kr)+B_{lj}(r_c)n_l(kr)\,, \quad r\geqslant r_c\,, \qquad (7)$$

for $\psi_{lj}(r)$ in the integrands on the right-hand side of Eq. (6) gives the first term of the solution of Eq. (3) expressed as a series in powers of the interaction $U_{lj}(r)$ in the region $r\geqslant r_c$. From Eqs. (1b) and (1c) it follows that this is a series in powers of the strength of the interaction $V_2(r)$.

In a scattering problem it is the asymptotic behavior of the partial wave $\psi_{lj}(r)$ that is of interest. To first order this is given as

$$\psi_{lj}(r)\sim j_l(kr)\{(1-b_{lj})A_{lj}(r_c)-c_{lj}B_{lj}(r_c)\} +n_l(kr)\{a_{lj}A_{lj}(r_c)+(1+b_{lj})B_{lj}(r_c)\}\,, \qquad (8)$$

where

$$a_{lj}=k\int_{r_c}^\infty dx\, x^2U_{lj}(x)[j_l(kx)]^2. \qquad (9)$$

The coefficients b_{lj} and c_{lj} are defined by replacing $[j_l]^2$ in Eq. (9) with $[j_ln_l]$ and $[n_l]^2$, respectively. The corresponding first-order phase shifts δ_{lj} are obtained from Eq. (8) as

$$\tan\delta_{lj}=\frac{(1+b_{lj})\tan\xi_{lj}-a_{lj}}{1-b_{lj}+c_{lj}\tan\xi_{lj}}\,, \qquad (10)$$

where

$$\tan\xi_{lj}=-B_{lj}(r_c)/A_{lj}(r_c)\,. \qquad (11)$$

The phase shifts ξ_{lj} describe the scattering from the short-range potential $V_1(r)$. We assume that the values of these phase shifts have been obtained—if necessary, by a numerical integration of the Schrödinger equation with interaction potential $V_1(r)$. It is worth noting that, since the coefficients a_{lj}, b_{lj}, and c_{lj} in Eq. (10) depend only on the interaction $V_2(r)$, the phase shifts ξ_{lj} contain all the information about the scattering from $V_1(r)$ that is necessary to the solution of the present problem. This is true also for all higher order approximations obtained by iteration of Eq. (6).

Frequently the long-range potential $V_2(r)$ has a simple radial dependence. For the special case

$$V_2(r)\propto U_{lj}(r)\propto r^{-n}\,, \quad n\geqslant 3\,, \qquad (12)$$

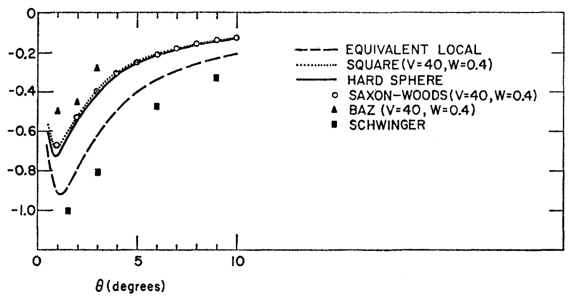

FIG. 1. A comparison of various calculations of the polarization of 1.0-MeV neutrons scattered from Pb as a function of the scattering angle. The respective strengths of the real and imaginary parts of the potential V_m associated with each calculation are denoted by V and W. The parameters for the equivalent local potential were obtained by the method discussed in Ref. 4. The Schwinger and Baz calculations are described in Refs. 1 and 3, respectively. For the hard-sphere calculation, Eq. (10) was used with r_c equal to the nuclear radius (8 F) and with the ξ_{lj} equal to the phase shifts for scattering from an impenetrable sphere of this radius.

the coefficients a_{lj}, b_{lj}, and c_{lj} for $l \geqslant 1$ can be evaluated by use of the relation[5]

$$(2l-1+n)\int_{r_c}^{\infty} dx\, x^{2-n} f_l(x) g_l(x)$$

$$=(2l+1-n)\int_{r_c}^{\infty} dx\, x^{2-n} f_{l-1}(x) g_{l-1}(x) + r_c^{3-n}[f_l(r_c)g_l(r_c)+f_{l-1}(r_c)g_{l-1}(r_c)], \quad (13)$$

where f_l and g_l are spherical Bessel functions, either j_l or n_l.

Since the phase shifts ξ_{lj} correspond to a potential of limited range, they become vanishingly small for sufficiently large values of l. Also, since $b_{lj} \ll 1$ for large l, there exists an l_0 such that for $l \geqslant l_0$ the δ_{lj} [Eq. (10)] approach the ordinary Born-approximation phase shifts δ_{lj}^B, where

$$\tan\delta_{lj}^B = -a_{lj}. \quad (14)$$

This result is of considerable practical value since it permits the use of the plane-wave Born approximation for the scattering amplitude as a device for summing the partial-wave series for large values of l. An example of the use of this device may be found in Ref. 2.

The fact that the phase shifts δ_{lj} must be independent of the value of the "cutoff" radius r_c provides a partial test of the accuracy of the approximation (10). For example, the values of the polarization calculated at any given angle by use of Eq. (10) (as discussed in Sec. III and IV) differ by less than two parts in 10^4 when r_c is changed from 15 to 25 F.

In the limit $r_c=0$, Eq. (1) becomes

$$V(r)=V_2(r) \quad r \geqslant r_c = 0, \quad (15)$$

and the entire potential energy of interaction is regarded as a perturbation. In this limit, Eq. (10) reduces to

$$\tan\delta_{lj} = -a_{lj}/(1-b_{lj}). \quad (16)$$

A Born approximation identical to Eq. (16) has been obtained by Brysk.[6] It has been shown by Falk[7] that this result is also obtained from the first-order Fredholm determinantal solution of the integral equation (3).

III. THE POLARIZATION CALCULATION

We now consider a calculation of the polarization of neutrons scattered from an optical-model potential $V_m(r)$, which may include a spin-orbit term, plus a potential $V_e(r)$ that describes the interaction between the magnetic moment of the neutron and the electric field of a charge Ze distributed uniformly over a sphere of radius r_s. The electromagnetic potential has the form

$$V_e(r) = [Ze^2\hbar^2|\mu_n|/(2m^2c^2)]\phi(r)\mathbf{l}\cdot\boldsymbol{\sigma}, \quad (17a)$$

where μ_n is the neutron magnetic moment (−1.9135 nuclear magnetons) and

$$\phi(r) = r_s^{-3} \quad \text{for} \quad r \leqslant r_s, \qquad = r^{-3} \quad \text{for} \quad r \geqslant r_s. \quad (17b)$$

The present calculations are found to be insensitive to the value of the radius r_s of the nuclear charge distribution. The "cutoff" radius r_c is chosen sufficiently large that $V_m(r)$ is negligibly small for $r \geqslant r_c$. The potentials $V_1(r)$ and $V_2(r)$ in Eqs. (1) then become

$$V_1(r) = V_m(r) + V_e(r) \quad \text{for} \quad r < r_c, \quad (18a)$$

and

$$V_2(r) = V_e(r) \quad \text{for} \quad r \geqslant r_c > r_s. \quad (18b)$$

By definition $V_1(r)$ vanishes for $r \geqslant r_c$ and $V_2(r)$ vanishes for $r < r_c$.

In the calculations reported here, the phase shifts ξ_{lj} for scattering from the potential $V_1(r)$ were evaluated by use of the ABACUS−2 program[8] modified to include the potential $V_e(r)$ in addition to the optical-model potential $V_m(r)$. The phase shifts δ_{lj} for scattering from the potential $V_1(r)+V_2(r)$ were then calculated by use of Eq. (10). Since $V_2(r)$ has the radial dependence given by Eq. (12) with $n=3$, the coefficients a_{lj}, b_{lj}, and c_{lj} in Eq. (10) can be evaluated by use of Eq. (13). For scattering angles much less than 1° the electron screening of the nuclear Coulomb field must be taken into account. This screening can be approximated by requiring finite values for the upper limit of the integrals that define these coefficients [Eq. (9)]. The necessary modification of Eq. (11) for this case can be obtained from Ref. 5.

[5] G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, London, 1944), 2nd ed., p. 136.

[6] H. Brysk, Phys. Rev. **126**, 1589 (1962); **133**, B1625 (1964).
[7] D. S. Falk, Phys. Rev. **129**, 2340 (1963).
[8] E. H. Auerbach, Brookhaven National Laboratory Report BNL-6562, 1962 (unpublished).

Once the values of the phase shifts δ_{lj} have been obtained, the polarization is calculated by means of the usual partial-wave series in terms of the associated Legendre polynomials P_l^1. We have used the relations given by Sample[2] to sum this series for large values of l.

Several calculations of the polarization of 1.0-MeV neutrons scattered from Pb through small angles are compared in Fig. 1. The open circles represent values calculated by use of Eq. (10) for an optical-model potential

$$V_m(r)=-(V_0+iW_0)\{1+\exp[(r-r_0)/a]\}^{-1}, \quad (19)$$

where $V_0+iW_0=(40.0+0.4i)$ MeV, $r_0=8$ F, and $a=0.5$ F. These points should be compared with the polarizations calculated by Baz[3] (solid triangles) for the same optical-model potential. These calculations differ only in that Baz's results are based on the usual generalization of the Born approximation discussed at the beginning of Sec. II. Baz takes the neighboring potential $V'(r)$[*Eq.* (2)] to be a square well. No nuclear spin-orbit interaction is included in any of the calculations shown in Fig. 1.

The dependence of the calculated polarization on the parameters chosen for the optical-model potential has been investigated (at least in a preliminary way). We consider the case of 1.0-MeV neutrons scattered from Pb and assume the optical potential to be of the Saxon-Woods form. For a surface absorption of strength $W_0=3.2$ MeV the polarization at angles less than 10° is increased by about 10% over values obtained for the volume absorption, Eq. (19), with $W_0=0.4$ MeV. An increase in the strength of the surface absorption (to 10 MeV) causes a further increase (by about 10%) in the polarization. The shape of the polarization curve as a function of scattering angle is not changed materially by these variations in the imaginary part of the optical-model potential. In fact, the angular dependence of the polarization seems to depend sensitively only on the value of r_0 in Eq. (19). An increase in the value of r_0 shifts the peak value of the polarization to smaller angles and also narrows the peak. For angles less than 10° the polarization is not sensitive to a variation of V_0.

IV. COMPARISON WITH POLARIZATION DATA

Figure 2 compares the results of the present calculation with the data of Elwyn *et al.*[4] In that work the magnitudes of the polarizations measured for 0.3- to 0.9-MeV neutrons scattered from Zr, Nb, Mo, and Cd through a laboratory angle of 24° were observed to be systematically larger than could be understood in terms of an optical-model potential alone, even after some parameter variation was attempted. The dashed curves in Fig. 2 represent calculations based on a potential equivalent to the nonlocal potential of Perey and Buck[9]

[9] F. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).

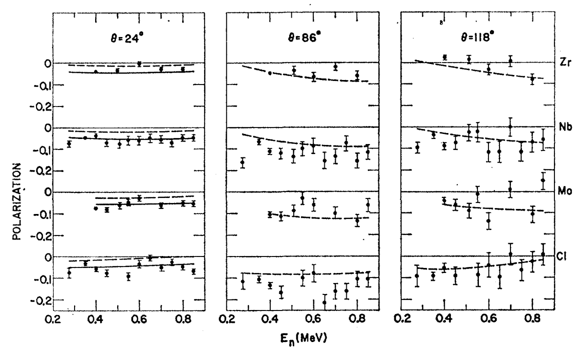

FIG. 2. A comparison of measured and calculated polarizations of neutrons scattered from four nuclei at three scattering angles as a function of neutron energy. The dashed curves represent optical-model calculations for a potential equivalent to the nonlocal potential of Perey and Buck plus a spin-orbit term of strength predicted by the shell model. The solid curve (at 24°) includes, in addition, the electromagnetic interaction $V_e(r)$. This latter interaction has a negligible effect on the polarization at the larger angles.

plus a spin-orbit potential of strength predicted by the shell model. The radial dependence assumed for the equivalent local potential is

$$V_m(r)=-V_Lf_s(r)-iW_Lf_D(r) +V_s(\hbar/m_\pi c)^2\boldsymbol{\sigma}\cdot\mathbf{l}(1/r)(d/dr)f_s(r), \quad (20a)$$

where

$$f_s(r)=[1+\exp\{(r-R)/a_s\}]^{-1}, \quad (20b)$$

and

$$f_D(r)=4\exp\left(\frac{r-R}{a_D}\right)\Big/\left[1+\exp\left(\frac{r-R}{a_D}\right)\right]^2. \quad (20c)$$

The values of the equivalent local parameters are given as a function of neutron energy in Table I of Ref. 4.

The polarizations calculated by use of the optical-model potential alone (the dashed curves in Fig. 2) are in quite good agreement with the values measured at 86 and 118° (as well as with the polarizations measured[4] at other angles), but the polarizations observed at 24° are consistently more negative than the calculated values. The solid curve in Fig. 2 represents a calculation in which the electromagnetic potential $V_e(r)$ is included in the total interaction. This calculation is based on Eq. (10). The total interaction potential is that defined by Eqs. (18), (17), and (20). In this latter calculation the agreement with measured values is systematically improved, and, as shown, the electromagnetic interaction accounts for a substantial part of the polarizations measured at 24°.

The interaction that results from the electric polarizability of the neutron can be included without difficulty in the approximation (10). This interaction gives an additional contribution to the differential cross section at small scattering angles. Calculations including this interaction are in progress.